MA 262 Review Exercises For Exam I – Solutions.

(1) $y' + \frac{4}{x} y = 12x$

$y(1) = 8$

Step 1: The integrating factor is $e^{\int \frac{4}{x} dx} = e^{4 \ln x} = e^{\ln x^4} = x^4$

So the equation becomes

$$\frac{d}{dx}(x^4 y) = x^4 \, 12x = 12x^5$$

Step 2: Integrate the equation: $x^4 y = 2x^6 + c$

$$y(x) = 2x^2 + cx^{-4}.$$

Since $y(1) = 8$; $y(1) = 2 + c = 8$ So $c = 6$

$$y(x) = 2x^2 + 6x^{-4}.$$

$$y(2) = 2.(2)^2 + \frac{6}{2^4} = 8 + \frac{6}{16} = 8 + \frac{3}{8} = \frac{67}{8}.$$

Answer C

2) $(y^2 + 2x + \cos x)\,dx + (2xy + \sin y)\,dy$

$M = y^2 + 2x + \cos x \,; \quad N = 2xy + \sin y$

$\frac{\partial M}{\partial y} = 2y \,; \quad \frac{\partial N}{\partial x} = 2y$

So $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Find F such that

$\frac{\partial F}{\partial x} = y^2 + 2x + \cos x\,; \quad \frac{\partial F}{\partial y} = 2xy + \sin y$

$F(x,y) = \int (y^2 + 2x + \cos x)\,dx + f(y)$

$= xy^2 + x^2 + \sin x + f(y)$

$\frac{\partial F}{\partial y} = 2xy + f'(y) = 2xy + \sin y$

So $f'(y) = \sin y$

$f(y) = -\cos y$

$F(x,y) = xy^2 + x^2 + \sin x - \cos y = C.$

when $x = 0$, $y = \pi$; $-\cos \pi = C$ $\quad C = 1$

$\boxed{xy^2 + x^2 + \sin x - \cos y = 1}$

when $x = \pi/2$ $\quad \frac{\pi}{2} Y^2 + \frac{\pi^2}{4} + 1 - \cos Y = 1$

(A) $\boxed{\frac{\pi}{2} Y^2 + \frac{\pi^2}{4} - \cos Y = 0}$

3) $$y' + \frac{2x}{1+x^2}\, y = -2xy^2$$

$$y(0) = 1.$$

Divide by y^2: $$y^{-2}y' + \frac{2x}{1+x^2}\, y^{-1} = -2x.$$

Set $v = y^{-1}$; $v' = -y^{-2}y'$

$$-v' + \frac{2x}{1+x^2}\, v = -2x$$

So $$v' - \frac{2x}{1+x^2}\, v = 2x,$$ Integrating factor

$$I = e^{-\int \frac{2x}{1+x^2}dx} = e^{-\ln(1+x^2)} = (1+x^2)^{-1}$$

So: $$\frac{d}{dx}\left((1+x^2)^{-1} v \right) = 2x\,(1+x^2)^{-1}$$

$$(1+x^2)^{-1} v = \ln(1+x^2) + c$$

$$v = \left(\ln(1+x^2) + c\right)(1+x^2)$$

$v(0) = (y(0))^{-1} = 1$ $\quad v(0) = c = 1.$ So

$$v(x) = (1+x^2)\left(1 + \ln(1+x^2)\right).$$

$v(1) = 2\,(1+\ln 2)$ $\qquad y(1) = \frac{1}{v(1)} = \frac{1}{2(1+\ln 2)}$

A

4) $\dfrac{dy}{dx} = \dfrac{x^3+4y^3}{3xy^2}$, $x>0$. (

Set $y/x = v$, $y = xv$; $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$v + x\dfrac{dv}{dx} = \dfrac{1+4v^3}{3v^2}$; $x\dfrac{dv}{dx} = \dfrac{1+4v^3}{3v^2} - v$

$x\dfrac{dv}{dx} = \dfrac{1+v^3}{3v^2}$ So: $\dfrac{3v^2}{1+v^3}\,dv = \dfrac{dx}{x}$.

$\displaystyle\int \frac{3v^2}{1+v^3}\,dv = \int \frac{dx}{x} + c$, So

$\ln|1+v^3| = \ln|x| + c$

$\ln|1+v^3| - \ln|x| = c$

$\ln\left|\dfrac{1+v^3}{x}\right| = c$ $\dfrac{1+v^3}{x} = c$.

$\dfrac{1+y^3/x^3}{x} = c$ $x^3 + y^3 = cx^4$.

$\boxed{y^3 + x^3 - cx^4 = 0}$

C

5) $yy'' = 3(y')^2$ (5)

$y(0) = 1, \quad y'(0) = 1$

Set $V = \frac{dy}{dx}$

$y'' = \frac{dV}{dx} = \frac{dV}{dy}\frac{dy}{dx} = V\frac{dV}{dy}$

$y \cdot V\frac{dV}{dy} = 3V^2$ So $\frac{dV}{V} = 3\frac{dy}{y}$.

$\ln|V| = \ln|y|^3 + c.$

$\boxed{V = c\,y^3}$ when $x = 0, y = 1$ and $V = y' = 1$

So $c = 1$

$V = y^3 = \frac{dy}{dx}$

$\int y^{-3}dy = \int dx + C_1 \qquad -\frac{1}{2}y^{-2} = x + C_1$

So $y^{-2} = C_1 - 2x$; $y(0)^{-2} = C_1 = 1$

$y^{-2} = 1 - 2x$; $y = \frac{1}{\sqrt{1-2x}}$

$y(3/8) = \frac{1}{\sqrt{1-3/4}} = \frac{1}{\sqrt{1/4}} = 2.$ B

6) $xy' = y + \frac{5}{4}(x^4 y)^{1/5}$

$y(1) = 1$

Divide by x: $y' = y/x + \frac{5}{4}\left(\frac{y}{x}\right)^{1/5}$

Set $y/x = v$ $\quad y = xv$, $y' = v + xv'$

$v + xv' = v + \frac{5}{4}(v)^{1/5}$ So $xv' = \frac{5}{4}(v)^{1/5}$

$\int \frac{4}{5} v^{-1/5}\, dv = \int \frac{dx}{x} + c$

$v^{4/5} = \ln x + c$

$\left(y/x\right)^{4/5} = \ln x + c$ Since $y(1) = 1$ $\quad c = 1$

$\left(y/x\right)^{4/5} = 1 + \ln x$

$y/x = (1 + \ln x)^{5/4}$

Since $y(1) = 1$, $y > 0$

So $\boxed{y = x(1 + \ln x)^{5/4}}$

A

7) Newton's Law says that if $T(t)$ (7)

Page 4 is the temperature of the object.

and M_0 is the temperature of the refrigerator:

$$\frac{dT}{dt} = K(M_0 - T(t))$$

$$\frac{dT}{M_0 - T} = K\,dt. \qquad \ln|M_0 - T(t)| = -Kt + C.$$

$$|M_0 - T(t)| = e^C \cdot e^{-Kt}$$

So $\boxed{M_0 - T(t) = (M - T(0))\, e^{-Kt}}$

$\underline{M=0:}$

$T(0) = 32$

$T(1) = 16$

$$T(t) = T(0)\, e^{-Kt}. \qquad T(1) = T_0\, e^{-K}$$

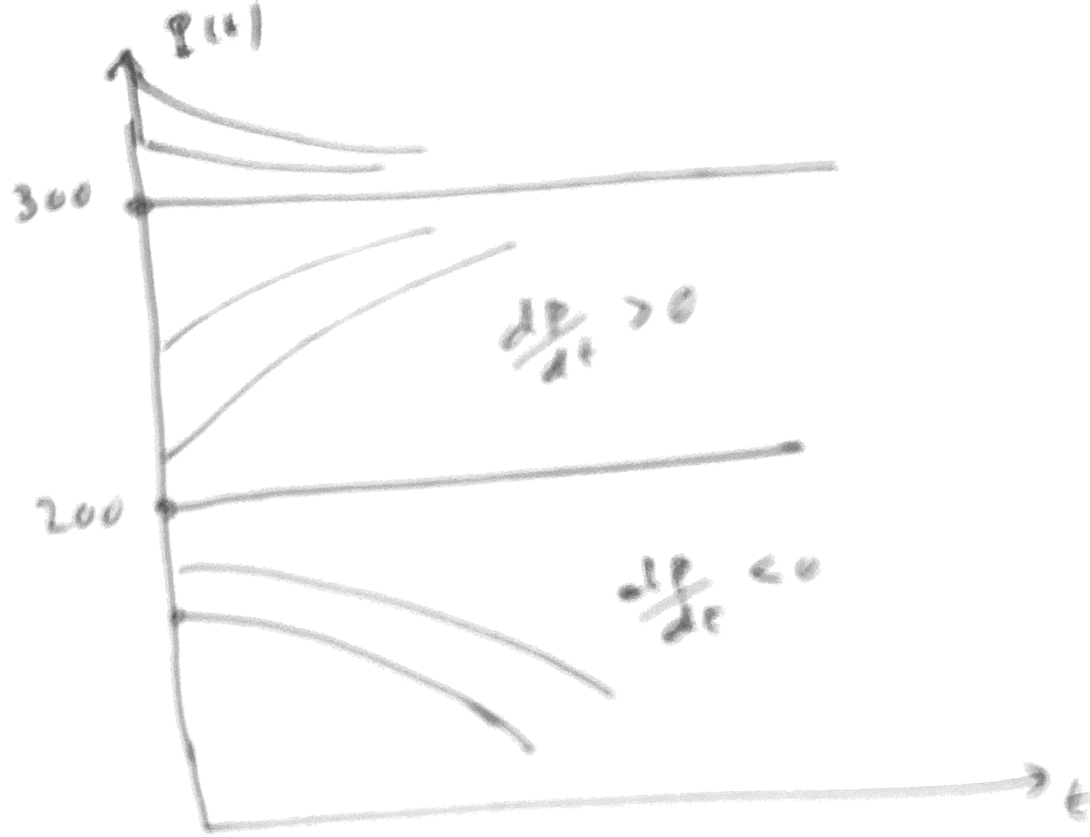
$$16 = 32\, e^{-K} \qquad \boxed{e^{-K} = 1/2}$$

when $t = 4$ $\quad T(t) = 32\, e^{-4K} = 32 \cdot \left(\tfrac{1}{2}\right)^4$

$$= \frac{32}{16} = 2.$$

B

8 Equilibrium Solution: $p(t) = 200$, $p(t) = 300$

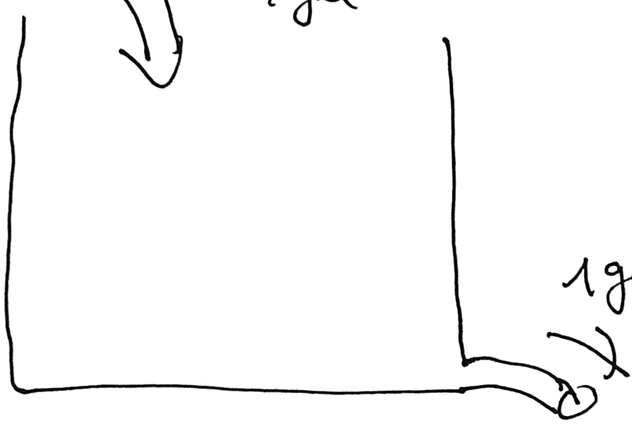


for $p_0 < 200$. $\frac{dp}{dt} < 0$

If $p(0) = 100$, $\frac{dp}{dt} < 0$ and eventually $p(t) = 0$ for some $t > 0$

E

9) $2\,lb/gal$ at $2\,gal/min$



$S(t)$ = amount of salt in the tank at time t

$1\,gal/min$ $V(t)$ = Volume of mixture in the tank

Initially $V(0)=100$, $S(0)=50$; $V(t)=100+t$

$$\frac{dS}{dt}=4-\frac{S(t)}{100+t}.$$ So

$$\frac{dS}{dt}+\frac{1}{100+t}S(t)=4 \qquad \frac{d}{dt}\big((100+t)S(t)\big)=4(100+t)$$

$$(100+t)S(t)=400t+2t^2+C.$$

$$S(t)=\frac{400t+2t^2+C}{100+t} \qquad S(0)=\frac{C}{100}=50$$

$$C=5000$$

$$S(t)=\frac{400t+2t^2+5000}{100+t} \qquad S(50)=\frac{30000}{150}$$

$$S(50)=\frac{3000}{15}=200\ lb$$ E

10)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 3 & 2 & 5 \\ 2 & 3 & k^2-2 & k+7 \end{array}\right] \xrightarrow[-2R_1+R_3]{-2R_1+R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 0 & +1 \\ 0 & 1 & k^2-4 & k+3 \end{array}\right] \xrightarrow{-R_1+R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 0 & +1 \\ 0 & 0 & k^2-4 & k+2 \end{array}\right]$$

The system has no solutions if $k^2-4=0$ but $k+2 \neq 0$. So $\boxed{K=2}$

B

#5, page 7

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & a \\ 2 & 2 & -3 & b \\ 3 & 1 & -5 & c \end{array}\right] \xrightarrow[-3R_1+R_3]{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & -1 & -2 & a \\ 0 & 4 & 1 & b-2a \\ 0 & 4 & 1 & c-3a \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & -1 & -2 & a \\ 0 & 4 & 1 & b-2a \\ 0 & 0 & 0 & (c-3a)-(b-2a) \end{array}\right]$$

$$c-3a-b+2a = c-b-a = 0 \qquad c = b+a.$$

12) Page 8 : $(2xy + x^3)\,dx + (x^2 + y^3 + 2)\,dy = 0$

$y(0) = 2.$

$M = 2xy + x^3; \quad \frac{\partial M}{\partial y} = 2x;$ So $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$N = x^2 + y^3 + 2, \quad \frac{\partial N}{\partial x} = 2x.$ and the equation is exact.

Find $F(x,y)$ such that

$\frac{\partial F}{\partial x} = M = 2xy + x^3$ and $\frac{\partial F}{\partial y} = N = x^2 + y^3 + 2.$

$F(x,y) = \int (2xy + x^3)\,dx + f(y)) = x^2 y + \frac{1}{4}x^4 + f(y)$

$\frac{\partial F}{\partial y} = x^2 + f'(y) = x^2 + y^3 + 2.$

So $f'(y) = y^3 + 2$

$f(y) = \frac{1}{4} y^4 + 2y$

$F(x,y) = x^2 y + \frac{1}{4}x^4 + \frac{1}{4}y^4 + 2y = c.$

$y(0) = 2 \qquad c = 4 + 4 = 8$

$$\boxed{x^2 y + \frac{1}{4}x^4 + \frac{1}{4}y^4 + 2y = 8}$$

C

13) <u>Page 9</u> : $xy' - 3y = x^3$

$y(1) = 1$

$y' - \frac{3}{x} y = x^2$ Integrating factor

$I(x) = e^{-\int 3/x \, dx} = x^{-3}$

$\frac{d}{dx}(x^{-3}y) = x^{-1}$ So $x^{-3}y = \ln x + c.$

$y(1) = c = 1$ So $y(x) = x^3 (1 + \ln x)$

So $\boxed{y(e) = 2e^3}$

14) <u>Page 10</u>. $2x^2yy' + 3x^2 + 2xy^2 = 0$

$$(2x^2y)dy + (3x^2 + 2xy^2)dx = 0$$

$$M(x,y) = 3x^2 + 2xy^2 \quad \frac{\partial M}{\partial y} = 4xy$$

$$N(x,y) = 2x^2y \quad , \quad \frac{\partial N}{\partial y} = 4xy$$

So this is an exact equah..

Find F such that $\frac{\partial F}{\partial x} = 3x^2 + 2xy^2$

$$\frac{\partial F}{\partial y} = 2x^2y$$

$$F(x,y) = \int (3x^2 + 2xy^2)\,dx \cancel{+f(y)} = x^3 + x^2y^2 + f(y)$$

$$\frac{\partial F}{\partial y} = 2yx^2 + f'(y) = 2x^2y$$

$f'(y) = 0$ so $f(y) = c$.

$$\boxed{x^3 + x^2y^2 = c.}$$

B.

10) Page 11.

$$3x + y - 5z = a$$
$$2x + 2y - 3z = b$$
$$x - y - 2z = c.$$

Write Augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & c \\ 2 & 2 & -3 & b \\ 3 & 1 & -5 & a \end{array}\right] \xrightarrow[-3R_1+R_3]{-2R_1+R_2}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & c \\ 0 & 4 & 1 & b-2c \\ 0 & 4 & 1 & a-3c \end{array}\right] \xrightarrow{-R_2+R_3}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & c \\ 0 & 4 & 1 & b-2c \\ 0 & 0 & 0 & a-b-c \end{array}\right]$$

If this is consistent, the system has a solution and so $a-b-c=0$

so $\boxed{c = a-b}$

15) Page 12:

$$xy' - y = x^2 e^x$$

So $y' - \frac{1}{x} y = x e^x$

$$\frac{d}{dx}(x^{-1}y) = e^x.$$

$x^{-1} y = e^x + c$ $\boxed{y = xe^x + cx.}$

A

16) Page 13: $(3x^2+y)\,dx + (x+2y)\,dy = 0.$

Find $F(x,y)$ $\frac{\partial F}{\partial x} = 3x^2+y$; $\frac{\partial F}{\partial y} = x+2y$

$F(x,y) = x^3 + xy + f(y)$; $x + f'(y) = x+2y$

$f'(y) = 2y$ so $f(y) = y^2.$

when $x=1, y=1$

$F(x,y) = x^3 + xy + y^2 = c$ So $c = 3$

$$\boxed{x^3 + xy + y^2 = 3}$$

A

17) Page 14 : $\frac{dy}{dx} = \frac{2x(y-2)}{x^2+1}$.

$$\frac{dy}{y-2} = \frac{2x}{x^2+1}\,dx$$

So $\ln|y-2| = \ln(1+x^2) + c$

$y - 2 = c(1+x^2)$. $y(0) - 2 = c$

$C = 2$.

$y = 2 + 2(1+x^2) = 2x^2+4$.

$\boxed{y(1) = 6}$ B

18) Page 15 $\left(\frac{2y}{x} + 2x\right)dx + (2\ln x - 3)\,dy = 0$

$M = 2y/x + 2x$, $\partial M/\partial y = 2/x$

$N = 2\ln x - 3$ $\partial N/\partial x = 2/x$

Find $F(x,y)$. $\frac{\partial F}{\partial x} = 2y/x + 2x$; $\frac{\partial F}{\partial y} = 2\ln x - 3$

$F(x,y) = 2y\ln x + x^2 + f(y)$; $2\ln x + f'(y) = 2\ln x - 3$

$f'(y) = -3$ $f(y) = -3y$.

$F(x,y) = 2y\ln x + x^2 - 3y = C$.

$\boxed{y(2\ln x - 3) + y^2 = C}$ A

19) <u>Page 16</u>

$$\left[\begin{array}{ccc|c} 3 & -2 & 5 & 1 \\ 0 & 2 & 1 & 1 \\ -3 & 6 & c & 1 \end{array}\right] \xrightarrow{R_1+R_3}$$

$$\left[\begin{array}{ccc|c} 3 & -2 & 5 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 4 & c+5 & 2 \end{array}\right] \xrightarrow{-2R_2+R_3}$$

$$\left[\begin{array}{ccc|c} 3 & -2 & 5 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & c+3 & 0 \end{array}\right].$$

The system has infinitely many solutions

if $c+3=0$ $\boxed{c=-3}$ A

It will have only one solution if $c+3 \neq 0$.

20) Page 17.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 8 & 4 \\ 2 & 3 & a^2-1 & a+1 \end{array}\right]$$

$$\xrightarrow{-R_2+R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 8 & 4 \\ 0 & 0 & a^2-9 & a-3 \end{array}\right]$$

the system is INCONSISTENT if it has no solutions. This happens

if $a^2-9=0$ but $a-3 \neq 0$

$\boxed{a=-3}$.

If $a^2-9=0$ and $a-3=0$

the system has infinitely many solutions

If $a^2-9 \neq 0$, the system has only one solution.

$\boxed{\text{E is the answer}}$

25) Pase [redacted] 18

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 1 & 2 & -1 & 4 \\ 3 & 1 & 2 & 7 \end{array}\right] \xrightarrow[-3R_1+R_3]{-R_1+R_2}$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 2 & -2 & 2 \\ 0 & 1 & -1 & 1 \end{array}\right] \xrightarrow[-1/2\, R_2+R_3]{+1/2\, R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_2 - x_3 = 1$$

$$x_1 + x_3 = 2$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2-x_3 \\ 1+x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \qquad x_3 = t$$

$$= \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

A

Page 19 # 7) $(x^2y+x)\,dy + (xy^2+y)\,dx = 0$

(Notice that the order is reversed). $M = xy^2+y$; $N = x+x^2y$

$\frac{\partial M}{\partial y} = 2xy+1$ and $\frac{\partial N}{\partial x} = 2xy+1$ so $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ and

the equation is exact. We want to find $F(x,y)$

such that $\frac{\partial F}{\partial x} = M = x^2y+y$; $\frac{\partial F}{\partial y} = N = x+x^2y$

$F(x,y) = \int (xy^2+y)\,dx + g(y) = \frac{1}{2}x^2y^2 + xy + g(y)$

So $\frac{\partial F}{\partial y} = yx^2 + x + g'(y) = x^2y+x$ $\quad g'(y)=0 \quad g=c$

So $F(x,y) = \frac{1}{2}x^2y^2+xy$ satisfies $\frac{\partial F}{\partial x} = M$, $\frac{\partial F}{\partial y} = N$.

The solution of the differential equation

satisfies $\boxed{\frac{1}{2}x^2y^2 + xy = C.}$ the curve goes through $(1,-1)$

So when $x=1, y=-1$ and we have $\frac{1}{2}-1 = C$, $C=-\frac{1}{2}$

$\frac{1}{2}x^2y^2 + xy = -\frac{1}{2}$ $\qquad x^2y^2+2xy = -1$

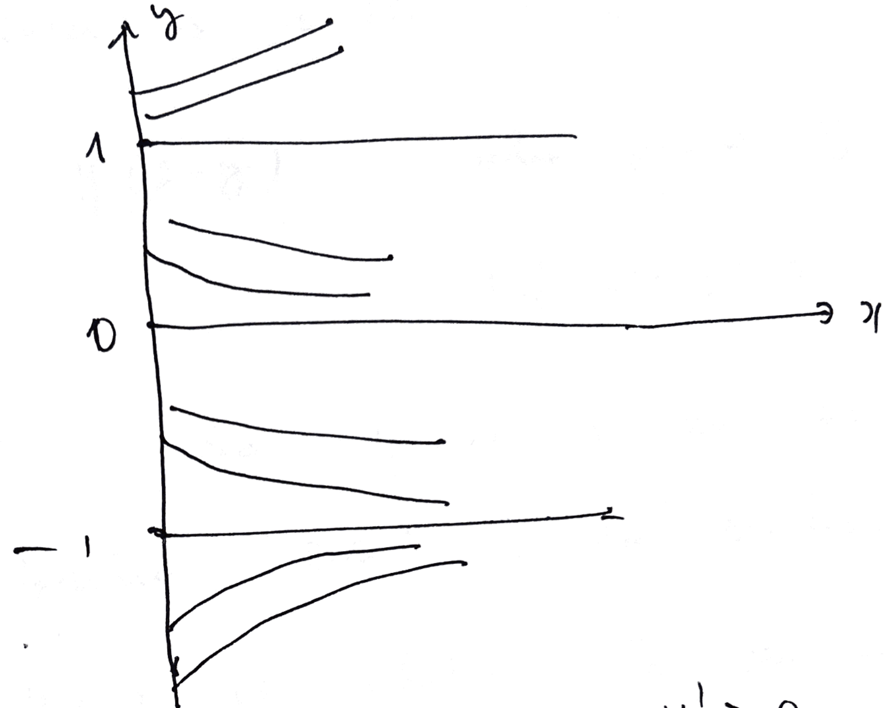
$x^2y^2+2xy+1 = 0$ $\qquad (xy+1)^2 = 0 \quad xy+1=0$

So the curve is $\boxed{y = -1/x}$ when $x=2$ $y=-\frac{1}{2}$.

B

Page 19 #8) $y' = y^2(y^2-1) = y^2(y-1)(y+1)$

So the equilibrium solutions are $y=0, y=1, y=-1$



when $y<-1$, $y^2(y-1)(y+1) > 0$ and so $y'>0$

So y is an increasing function

when $-1<y<0$; $y^2(y+1)(y-1) < 0$ so $y'<0$

and y is a decreasing function

So $y=-1$ is a stable equilibrium solution

when $0<y<1$ $y^2(y+1)(y-1) < 0$ so $y'<0$

$y=0$ is a semi stable equilibrium ~~position~~ solution.

when $y>1$ $y^2(y-1)(y+1) > 0$ and $y'>0$

so $y=1$ is an unstable eq solution

A

Page 20 : # 11) The equilibrium solutions are

$y=0$ and $y=3$. This alone says

that the answer is either A or C.

Item A : $y' = y(3-y)$ when $y<0$ $y' < 0$.

If $0<y<3$

$y' = y(3-y) > 0$ So $y=0$ is an unstable

equilibrium solution. The picture

Shows that $y=0$ is stable. So the answer

is not A.

Item D $y' = y(y-3)$ when $y<0$, $y'>0$

If $0<y<3$ $y' = y(y-3) < 0$

So $y=0$ is a stable solution.

when $y>3$ $y' = y(y-3) > 0$ and $y=3$

is unstable. This agrees with the

picture.

D

#6, Page 21

$$y' + y = y^2 e^x$$

$$y(0) = 1$$

Divide by y^2: $y^{-2}y' + y^{-1} = e^x$ and set $v = y^{-1}$

$\frac{dv}{dx} = -y^{-2}\frac{dy}{dx}$ so the equation becomes

$$-\frac{dv}{dx} + v = e^x \quad \text{and so}$$

$$\frac{dv}{dx} - v = -e^x.$$ The integrating factor is $e^{\int -x\,dx} = e^{-x}$ and the equation becomes

$$\frac{d}{dx}(e^{-x}v) = -e^{-x}\cdot e^x = 1$$

$$\frac{d}{dx}(e^{-x}v) = -1 \quad \text{So}$$

$$e^{-x}v = -x + c$$

$$\boxed{v = e^x(c-x)}$$ But $v = \frac{1}{y}$

$$y = \frac{1}{e^x(c-x)} = \frac{e^{-x}}{c-x}$$

$y(0) = 1 = \frac{1}{c}$

So $y(x) = \dfrac{e^{-x}}{1-x}$

$y(1/2) = \dfrac{e^{-1/2}}{1-1/2} = 2e^{-1/2}$

Page 22 $\quad y'' + 2y^{-1}(y')^2 = y'$

$y(0) = 1$ and $y'(0) = \frac{1}{3}$

Step 1: Set $v = y'$ and think of

$v(x) = v(y(x))$ so $\frac{dv}{dx} = y'' = \frac{dv}{dy}\frac{dy}{dx}$

$= v\frac{dv}{dy}$.

The equation becomes: $v\frac{dv}{dy} + \frac{2}{y}v^2 = v$

Since $v = y'$ and so $v = \frac{1}{3}$ when $y = 1$.

Divide by v $\quad \frac{dv}{dy} + \frac{2}{y}v = 1$

$\frac{d}{dy}(y^2 v) = y^2 \qquad y^2 v = \frac{1}{3}y^3 + c.$

$v = \frac{1}{3}y + cy^{-2} \qquad$ when $y = 1$: $v = \frac{1}{3} + c = \frac{1}{3}$

so $c = 0$

$v = \frac{1}{3}y = \frac{dy}{dx}$: $\quad y = c\,e^{\frac{1}{3}x}$.

Since $y(0) = 1 \qquad y = e^{\frac{1}{3}x} \qquad \boxed{y(3) = e}$

E

Page 23: We want to find the reduced row echelon form of $A = \begin{bmatrix} 3 & 9 & 1 \\ 2 & 6 & 7 \\ 1 & 3 & -6 \end{bmatrix}$

Step 1: Switch the 1st and 3rd rows

$$\begin{bmatrix} 1 & 3 & -6 \\ 2 & 6 & 7 \\ 3 & 9 & 1 \end{bmatrix} \xrightarrow[-3R_1+R_3]{-2R_1+R_2} \begin{bmatrix} 1 & 3 & -6 \\ 0 & 0 & 19 \\ 0 & 0 & 19 \end{bmatrix}$$

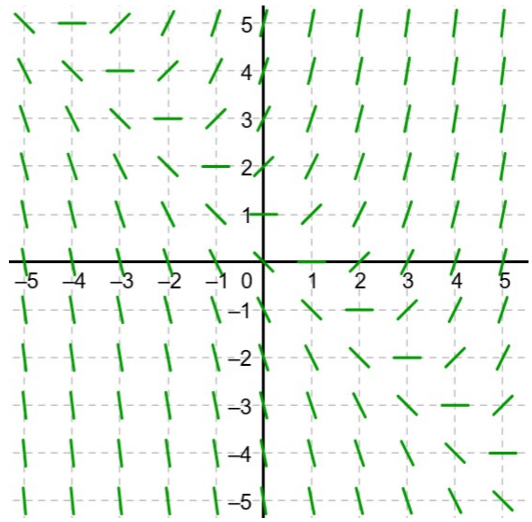
$$\xrightarrow{-R_2+R_3} \begin{bmatrix} 1 & 3 & -6 \\ 0 & 0 & 19 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2/19}$$

$$= \begin{bmatrix} 1 & 3 & -6 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow[+6R_2+R_1]{} \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

I. is false II is correct. III is correct

IV is false

1. The slope field shown below is the graphical representation of which of the following differential equations?



A. $\dfrac{dy}{dx} = x + y - 1$

B. $\dfrac{dy}{dx} = x - y - 1$

C. $\dfrac{dy}{dx} = y - x - 1$

D. $\dfrac{dy}{dx} = x + y$

E. $\dfrac{dy}{dx} = y - x$

This is the only one which is consistent with the facts that dy/dx=-1 when x=-y

Solution. #12) Let $v = x+y$

$$\frac{dv}{dx} = 1 + \frac{dy}{dx} \quad \text{So} \quad \frac{dy}{dx} = \frac{dv}{dx} - 1$$

$$\frac{dv}{dx} - 1 = -\sin^2 v \qquad \frac{dv}{dx} = 1 - \sin^2 v = \cos^2 v$$

$$\frac{dv}{\cos^2 v} = dx \qquad \sec^2 v \, dv = dx$$

$$\tan v = x + c \qquad v = \tan^{-1}(x+c)$$

$$\text{So} \quad y = \tan^{-1}(x+c) - x \qquad y(0) = \tan^{-1}(c) = \pi/4$$

$$c = 1$$

$$\boxed{y = \tan^{-1}(x+1) - x}$$